Digital Communications

« Linear » Carrier Modulations

« The complex envelop associated to the transmitted signal linearly depends on the message »

1) One or two dimensional modulations
2) Complex envelop
3) Equivalent lowpass channel
4) Performance

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Baseband Modulation

**One-dimensional**

**M-ASK (Amplitude Shift Keying)**

Binary information: 0 1 1 0 0

Baseband Modulation: $m(t) = \sum_k a_k h(t - kT_s)$

Frequency transposition: $\cos(2\pi f_p t)$

Down conversion

Carrier-modulated signal: Spectrum around a carrier frequency $f_p$

LPF

Coherent demodulation

Retrieved Binary information: 0 0 1 0 1

Baseband Demodulation
Linear Carrier Modulation
One-dimensionnal

M-ASK (Amplitude Shift Keying)

Example: 4-ASK, rectangular shaping

\[ m(t) = \sum_{k} a_k h(t - kT_s) \]

Signal modulated on \( f_p \):

\[ x(t) = \sum_{k} a_k h(t - kT_s) \cos(2\pi f_p t) \]

\[ S_m(f) = 5T_s \text{sinc}^2(\pi f T_s) \]

\[ S_x(f) = \frac{5T_s}{4} \left\{ \text{sinc}^2(\pi(f - f_p)T_s) + \text{sinc}^2(\pi(f + f_p)T_s) \right\} \]
Linear Carrier Modulation

Two-dimensional

Baseband Modulation

\[ x(t) = \sum_{k} a_k h(t - kT_s) + \cos(2\pi f_p t) \]

\[ \sum_{k} b_k h(t - kT_s) + \sin(2\pi f_p t) \]

Downconversion

Baseband Demodulation

Binary information:

0 1 1 0 0

Information binaire:

1 0 1 1 0 0

Coherent demodulation
Orthogonal signals

LPF

LPF
Linear Carrier Modulation

Complex envelop

Binary information: 0 1 1 0 0

Baseband Modulation

\[ x(t) = \sum_k a_k h(t - kT_s) \cos(2\pi f_p t) - \sum_k b_k h(t - kT_s) \sin(2\pi f_p t) \]

I(t) In Phase Component

Q(t) Quadrature Component

\[ x(t) = \Re \left[ (I(t) + jQ(t)) e^{j2\pi f_p t} \right] \]

\[ x_e(t) = I(t) + jQ(t) = \sum_k d_k h(t - kT_s) \]

Complex envelop associated to x(t)
Linear Carrier Modulation

Complex envelop

Baseband Modulation

Binary information: 01100

Baseband Modulation

\[ x(t) = \sum_k a_k h(t - kT_s) \cos(2\pi f_p t) - \sum_k b_k h(t - kT_s) \sin(2\pi f_p t) \]

Bits → Mapping → \( \{d_k\} \) → \[ \sum_k d_k \delta(t - kT_s) \] → \( h(t) \) → \( x_e(t) = \sum_k d_k h(t - kT_s) \) → \( \Re[.] \) → \( x(t) \)

Complex symbols

\( d_k = a_k + jb_k \)

Complex baseband modulation

Complex envelop associated to \( x(t) \):

\[ x_e(t) = I(t) + jQ(t) \]

Frequency transposition

\[ x(t) = \Re\left[ x_e(t) e^{j2\pi f_p t} \right] \]
Linear Carrier Modulation

Complex envelop

Bits → Mapping → \{d_k\} → \sum_k d_k \delta(t - kT_s) → h(t) → x_e(t) = \sum_k d_k h(t - kT_s)

Complex symbols
\[ d_k = a_k + j b_k \]

Complex envelop associated to \( x(t) \):
\[ x_e(t) = I(t) + j Q(t) \]

Complex message generation with a baseband modulator

→ The PSD of the carrier-modulated signal:
\[ x(t) = \Re [x_e(t)e^{j2\pi f_p t}] \rightarrow R_x(\tau) = \frac{1}{2} \Re \left[ R_{x_e}(\tau)e^{j2\pi f_p \tau} \right] \rightarrow S_x(f) = \frac{1}{4} (S_{x_e}(f - f_p) + S_{x_e}(-f - f_p)) \]

is obtained from the PSD of its associated complex envelop (known baseband spectrum):
\[ S_x(f) = \frac{\sigma_d^2}{T_s} |H(f)|^2 + 2\frac{\sigma_d^2}{T_s} |H(f)|^2 \sum_{k=1}^{\infty} \Re \left[ R_d(k)e^{j2\pi f k T_s} \right] + \frac{|m_d|^2}{T_s^2} \sum_k \left| H \left( \frac{k}{T_s} \right) \right|^2 \delta \left( f - \frac{k}{T_s} \right) \]

Re-use the results obtained for baseband modulations
Linear Carrier Modulation

Two main classes of two-dimensionnal modulations

- **Complex baseband modulation**
  - Bits $\rightarrow$ Mapping $\rightarrow \{d_k\} \rightarrow \sum_k d_k \delta(t - kT_s) \rightarrow h(t) \rightarrow x_e(t) = \sum_k d_k h(t - kT_s)$
  - Complex symbols $d_k = a_k + j b_k$
  - Complex envelop associated to $x(t)$:
    $$x_e(t) = I(t) + jQ(t)$$

- **Frequency transposition**
  - $x(t) = \Re[\exp(2\pi f_p t)]$

→ $\{a_k\}$ and $\{b_k\}$ M-ary independent symbols $\in \{\pm1, \pm3, ..., \pm(\sqrt{M} - 1)\}$

- **square M-QAM (Quadrature Amplitude Modulation)**
  - $d_k \in \{e^{j\left(\frac{2\pi l}{M} + \frac{\pi}{M}\right)}\}$, $l = 0, ..., M - 1$

- **M-PSK (Phase Shift Keying)**
Let’s assume that the symbols are independent, equally likely and with a zero mean. For a given bit rate $R_b$, a transmission using a 8-PSK modulation and a rectangular shaping filter will be more spectrally efficient than:

A. a transmission using a QPSK modulation with the same rectangular shaping filter.

B. a transmission using a 8-PSK modulation with a square root raised cosine shaping filter.

C. a transmission using a 16-QAM modulation with the same rectangular shaping filter.
A 4-state phase modulation:
- A: TRUE
- B: FALSE

A 4-state QAM modulation:
- A: TRUE
- B: FALSE

Less spectrally efficient than a BPSK modulation using the same shaping filter:
- A: TRUE
- B: FALSE

More robust, for the same level of noise and the same transmitted power, than a BPSK modulation:
- A: TRUE
- B: FALSE
Linear Carrier Modulation

**Constellation**

Representation of possible $d_k$'s in the $(a_k, b_k)$ plane = « constellation » of the modulation

**QAM Constellations**
- Power efficient
- (DVB-C, DVB-T, xDSL)

**PSK Constellations**
- Robust to non linearities
- (DVB-S)

**Hybrid modulations : APSK**
- (DVB-S2, DVB-S2X)
Linear Carrier Modulation

Examples

→ Two-dimensionnal linear modulations: M-QAM

Example: 4-QAM or QPSK (DVB-S)

<table>
<thead>
<tr>
<th>bits</th>
<th>$a_k$</th>
<th>$b_k$</th>
<th>$d_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>-1</td>
<td>-1</td>
<td>-1-j</td>
</tr>
<tr>
<td>01</td>
<td>-1</td>
<td>+1</td>
<td>-1+j</td>
</tr>
<tr>
<td>10</td>
<td>+1</td>
<td>-1</td>
<td>1-j</td>
</tr>
<tr>
<td>11</td>
<td>+1</td>
<td>+1</td>
<td>1+j</td>
</tr>
</tbody>
</table>

$I(t)$ and $Q(t)$

$x(t)$

Independent \{$a_k$\} and \{$b_k$\}
→ Two-dimensional linear modulations: M-QAM

Example: 16-QAM (DVB-C)

<table>
<thead>
<tr>
<th>Bits</th>
<th>0000</th>
<th>0001</th>
<th>...</th>
<th>1110</th>
<th>1111</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_k$</td>
<td>+3</td>
<td>+3</td>
<td></td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$b_k$</td>
<td>+3</td>
<td>+1</td>
<td></td>
<td>-3</td>
<td>-1</td>
</tr>
<tr>
<td>$d_k$</td>
<td>3+3j</td>
<td>3+j</td>
<td></td>
<td>-1-3j</td>
<td>-1-j</td>
</tr>
</tbody>
</table>

$x(t)$

$I(t)$

$Q(t)$

$a_k$

$b_k$
Two-dimensionnal linear modulations: M-PSK

Example: 8-PSK (DVB-S2)

\[ d_k \in \{ e^{j\left(\frac{2\pi k}{8}l + \frac{\pi}{8}\right)} \}, \ l = 0, \ldots, 7 \]
Linear Carrier Modulation

Examples

→ Hybrid modulations: M-APSK (DVB-S2)

16-QAM

16-APSK (4-12 APSK)

32-APSK (4-12-16 APSK)
Linear Carrier Modulation

Examples

→ **Hierarchical modulations**: DVB-T and T2, DVB-H, DVB-S2

Example 1: hierarchical 16-QAM (DVB-T or H)

Example 2: hierarchical 8-PSK (DVB-S2)
Linear Carrier Modulation

Transmitter

Bits $\rightarrow$ Mapping $\{d_k\}$ $\rightarrow$ $\sum_k d_k\delta(t - kT_s)$ $\rightarrow$ $h(t)$ $\rightarrow$ $x_e(t) = \sum_k d_k h(t - kT_s)$

Complex symbols $d_k = a_k + jb_k$

Complex baseband modulation

$\rightarrow$ M-ASK: $d_k = a_k \in \{\pm1, \ldots, \pm(M - 1)\}$

$x_e(t) = I(t)$

$x(t) = I(t) \cos(2\pi f_p t)$

$\rightarrow$ M-QAM: $d_k = a_k + jb_k$ $\text{avec } a_k \text{ et } b_k \in \{\pm1, \ldots, \pm(\sqrt{M} - 1)\}$

$\rightarrow$ M-PSK: $d_k \in \{e^{j\left(\frac{2\pi l}{M} + \frac{\pi}{M}\right)}\}, \ l = 0, \ldots, \ M - 1$

$x_e(t) = I(t) + jQ(t)$

$x(t) = I(t) \cos(2\pi f_p t) - Q(t) \sin(2\pi f_p t)$

Frequency transposition

\[e^{j2\pi f_p t}\]
Linear Carrier Modulation

Receiver

Mapping\^{-1}

\rightarrow \hat{d}_m

Decisions

\leftarrow z(t_0 + mT_s)

\rightarrow \hat{a}_m

\rightarrow \hat{b}_m

\rightarrow \hat{d}_m

Mapping

\rightarrow \hat{d}_m

Decisions

\leftarrow \hat{a}_m

\leftarrow \hat{b}_m

\leftarrow \hat{d}_m

M-ASK:

M-QAM:

M-PSK:

Sampling

\rightarrow z(t)

Receiver filter

h_r(t)

Baseband Demodulation

Downconversion

LPF

\rightarrow \cos(2\pi f_p t)

LPF

\rightarrow \sin(2\pi f_p t)
Linear Carrier Modulation

**Equivalent lowpass channel to reduce the processing time for digital implementations**

**Baseband modulation**

- Bits → Mapping → \( \{d_k\} \) → \( \sum_k d_k \delta(t - kT_s) \) → \( h(t) \) → \( x_e(t) = \sum_k d_k h(t - kT_s) \)

  Complex envelop associated to \( x(t) \):
  \[ x_e(t) = I(t) + jQ(t) \] (Bande \( B_e \))

**Frequency transposition**

- \( e^{j2\pi f_c t} \)

**Transmission channel**

- \( h_c(t) \)

**Downconversion**

- \( \cos(2\pi f_p t) \)
- \( \sin(2\pi f_p t) \)

**Example of used bandwidth for satellite broadcasting:**

- L: 1.4-1.6 GHz, C: 4-6 GHz, Ku: 10.70-12.75 GHz, Ka: 20-30 GHz.

**Baseband Demodulation**

- Demapping → \( \{\hat{d}_m\} \) → Decisions → Sampling → \( z_m \) → \( z(t) \) → Receiver filter \( h_r(t) \)

- \( t_0 + mT_s \)
Linear Carrier Modulation

Equivalent lowpass channel to reduce the processing time for digital implementations

- **Bits** → **Mapping** → $\{d_k\} \rightarrow \sum_k d_k \delta(t - kT_s)$ → **$h(t)$** → $x_e(t) = \sum_k d_k h(t - kT_s)$
  - Complex envelop associated to $x(t)$:
    - $x_e(t) = I(t) + jQ(t)$ (Bande $B_e$)
  - Baseband modulation

- **$F_{\text{max}} = B_e$**
  - **Lower sampling frequencies**

- **Demapping** → **Decisions** → **Sampling** → **$z_m$** → **$z(t)$** → **Receiver filter $h_r(t)$**
  - Downconversion
    - **Low pass**
      - $\cos(2\pi f_{\text{pf}} t)$
    - **Low pass**
      - $\sin(2\pi f_{\text{pf}} t)$

- **Baseband Demodulation**
  - **Band pass filter**
Linear Carrier Modulation

Complex envelop

Complex message generation with a baseband modulator

Bits $\rightarrow$ Mapping $\rightarrow \{d_k\} \rightarrow \sum_k d_k \delta(t - kT_s) \rightarrow h(t) \rightarrow x_e(t) = \sum_k d_k h(t - kT_s)$

Complex symbols $d_k = a_k + jb_k$

Complex envelop associated to $x(t)$:

$x_e(t) = I(t) + jQ(t)$

Frequency transposition

$\exp(2\pi f_p t)$

$\rightarrow$ PSD of the carrier-modulated signal:

$S_x(f) = \frac{1}{4} \{S_{x_e}(f - f_p) + S_{x_e}(-f - f_p)\}$

$S^+_x(f)$

$S^-_x(f)$

$-f_p$

$f_p$

$\rightarrow$ PSD of the corresponding complex envelop:

$S_{x_e}(f) = 4S^+_x(f + f_p)$

(or also $X_e(f) = 2X^+_e(f + f_p)$ for deterministic signals)
Complex envelop associated to the bandpass channel:

\[ h_c(t) = I_c(t) + jQ_c(t) \]

\[ H_c(f) = \frac{1}{2} (H_c(f - f_p) + H_c(f + f_p)) \]

(remark: the channel is assumed to be ideal in the figure)
Linear Carrier Modulation
Equivalent lowpass channel: construction

→ Bandpass filtering:

\[
S_x(f) = \frac{1}{4}(S_{x_e}(f - f_p) + S_{x_e}(-f - f_p))
\]

→ Complex envelop associated to the filtered noise:

\[
b_e(t) = I_b(t) + jQ_b(t)
\]

\[
S_{I_b}(f) = S_{Q_b}(f) = S_{b_e}^+(f - f_p) + S_{b_e}^-(f + f_p) = N_0
\]

\[
S_b(f) = \frac{1}{4}(S_{b_e}(f - f_p) + S_{b_e}(-f - f_p))
\]
Linear Carrier Modulation

Equivalent low pass channel: construction

**Baseband modulation**

- **Bits** → **Mapping** \(\{d_k\}\) → \(\sum_k d_k \delta(t - kT_s)\) → **h(t)** → **\(x_e(t) = \sum_k d_k h(t - kT_s)\)**
  - Symboles complexes
  - \(d_k = a_k + j b_k\)

**Frequency transposition**

- Complex envelop associated to **\(x(t)\)**: **\(x_e(t) = I(t) + j Q(t)\)** (Bande \(B_e\))
- **\(e^{j2\pi f_H t}\)**

**Downconversion**

- **\(h_{c_e}(t) / 2\)**
- **\(b_e(t) = I_b(t) + j Q_b(t)\)**

**Baseband Demodulation**

- **Bits** → **Demapping** \(\{\hat{d}_m\}\) → **Decisions** → **Sampling** \(z_m\) → **Receiver filter** \(h_r(t)\) → **\(z(t)\)** → **Low pass filter**
  - **cos \(2\pi f_{pk}\)**
  - **sin \(2\pi f_{pt}\)**
Linear Carrier Modulation
Equivalent lowpass channel

Mapping $\{d_k\} \rightarrow \sum_k d_k \delta(t - kT_s) \rightarrow h(t) \rightarrow x_e(t) = \sum_k d_k h(t - kT_s)$

Complex envelope associated to $x(t)$:
$x_e(t) = I(t) + jQ(t)$ (Bande $B_e$)

Symboles complexes $d_k = a_k + j b_k$

Baseband modulation

Fullfill Nyquist criterion on:
$g_e(t) = h(t) \ast \frac{h_{ce}(t)}{2} \ast h_r(t)$

Equivalent lowpass channel

$g_e(t) \rightarrow h_{ce}(t) \rightarrow \frac{h_{ce}(t)}{2} \rightarrow b_e(t) = I_b(t) + jQ_b(t)$

Baseband Demodulation

Demapping $\{\hat{d}_m\} \rightarrow$ Decisions $z_m$ $\rightarrow$ Sampling $z(t)$ $\rightarrow$ Receiver filter $h_r(t)$

Matched filtering

$z(t) \rightarrow t_0 + mT_s$ $\rightarrow$ $h_r(t) = \lambda h_e^*(t_0 - t)$, avec $h_e(t) = h(t) \ast \frac{h_{ce}(t)}{2}$

Baseband SER computations can be re-used
Linear Carrier Modulation

Performance (Hypothesis: Nyquist + Matched filtering)

→ **M-ASK**

Bits $\rightarrow$ Mapping $\rightarrow$ $\{d_k\} \rightarrow \sum_k d_k \delta(t - kT_s) \rightarrow h(t) \rightarrow \frac{I_c(t)}{2} \rightarrow h_r(t) \rightarrow \text{Mapping}^{-1} \rightarrow$ Bits

\[ d_k = a_k \in \{\pm1, \ldots, \pm(M-1)\} \]

\[ \text{SER} = \text{SER}_I = 2 \left(1 - \frac{1}{M}\right) Q \left(\sqrt{\frac{6 \log_2(M) E_b}{M^2 - 1} N_0}\right) \]

→ **Squared M-QAM**

\[ d_k = a_k + jb_k \text{ with } a_k \text{ and } b_k \in \{\pm1, \ldots, \pm(\sqrt{M} - 1)\} \]

\( \Leftrightarrow \) two independent $\sqrt{M}$-PAM transmissions

But !! \( E_s = \text{physical parameter} = \text{average symbol energy at the receiver input (M symbols } d_k) \) !!

\[ \text{SER} \approx 2\text{SER}_I = 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{3 E_s}{M - 1} N_0}\right) = 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{3 \log_2(M) E_b}{M - 1} N_0}\right) \]

→ **M-PSK**

Bits $\rightarrow$ Mapping $\rightarrow$ $\{d_k\} \rightarrow \sum_k d_k \delta(t - kT_s) \rightarrow h(t) \rightarrow \frac{h_{c_2}(t)}{2} \rightarrow h_r(t) \rightarrow \text{Arg}[.] \rightarrow \text{Mapping}^{-1} \rightarrow$ Bits

\[ d_k \in \{e^{j(\frac{2\pi l}{M} + \frac{\pi}{M})}, \ l = 0, \ldots, M-1\} \]

\[ \text{SER} = 2Q \left(\sqrt{\frac{2E_s}{N_0} \sin \left(\frac{\pi}{M}\right)}\right) \]
Linear Carrier Modulation

BER comparison for M-QAM and M-PSK

PSK (dotted) vs QAM (plain)

Power efficiency for PSK
Same spectral efficiency
QUESTION
Assuming, for each case, that the shaping filter is the same and that the transmission channel is optimized (Gray Mapping, Nyquist, Matched filtering, optimal sampling and thresholds), a modulation 16-QAM will be:

More power efficient than a 16-PSK modulation:
- A TRUE
- B FALSE

More spectrally efficient than a 16-PSK modulation:
- A TRUE
- B FALSE

More power efficient than a QPSK modulation:
- A TRUE
- B FALSE

More spectrally efficient than a QPSK modulation:
- A TRUE
- B FALSE
Assuming, for each case, that transmission channel is optimized (Gray Mapping, Nyquist, Matched filtering, optimal sampling and thresholds), using a 16-QAM modulation with a rectangular shaping filter is:

More power efficient than using a 16-QAM modulation with a square root raised cosine filter:
- A TRUE
- B FALSE

More spectrally efficient than using a 16-QAM modulation with a square root raised cosine filter:
- A TRUE
- B FALSE
Example of physical layer on an AWGN channel:

Satellite Digital Video Broadcasting: DVB-S

Physical layer

Source coding and multiplexing

- Program 1
  - Video Coder
  - Audio Coder
  - Data Coder
  - PCR
  - STC 1

- Program N
  - Video Coder
  - Audio Coder
  - Data Coder
  - PCR
  - STC N

Program Information

Transport Stream

MPEG-2 System Transport stream generation

- Reed Solomon
  - RS(204,188, t=8)
- Forney Convolutional Interleaving
- Convolutional Code (7,1/2)
- QPSK
- SRRCF
  - $\alpha = 0.35$

To RF Satellite Channel

Mux Adaptation and energy dispersal

- Outer code
  - Reed Solomon
  - RS(204,188, t=8)
  - Forney Convolutional Interleaving

Table D.1: Example of System performance over 33 MHz transponder

<table>
<thead>
<tr>
<th>Bit Rate $R_u$ (after MUX) [Mbit/s]</th>
<th>Bit Rate $R'_u$ (after RS) [Mbit/s]</th>
<th>Symbol Rate [Mbaud]</th>
<th>Convolut. Inner Code Rate</th>
<th>RS Outer Code Rate</th>
<th>C/N (33 MHz) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23,754</td>
<td>25,776</td>
<td>25,776</td>
<td>1/2</td>
<td>188/204</td>
<td>4.1</td>
</tr>
<tr>
<td>31,672</td>
<td>34,368</td>
<td>25,776</td>
<td>2/3</td>
<td>188/204</td>
<td>5.8</td>
</tr>
<tr>
<td>35,631</td>
<td>38,664</td>
<td>25,776</td>
<td>3/4</td>
<td>188/204</td>
<td>6.8</td>
</tr>
<tr>
<td>39,590</td>
<td>42,960</td>
<td>25,776</td>
<td>5/6</td>
<td>188/204</td>
<td>7.8</td>
</tr>
<tr>
<td>41,570</td>
<td>45,108</td>
<td>25,776</td>
<td>7/8</td>
<td>188/204</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Digital TV transmission must be « Quasi Error Free » (QEF) TE $< 10^{-10}$
Satellite Digital Video Broadcasting: DVB-S
Physical layer

Mux Adaptation and energy dispersal

Codage source et multiplexage

Program 1
Video Coder
Audio Coder
Data Coder
STC 1

ES
PES
PCR

Train transport

Program N
Video Coder
Audio Coder
Data Coder
STC N

Program Information

Example on an image

Scrambling

PSD of the unscramble signal:

PSD of the scramble signal:

Fréquences normalisées

Partie positive de la DSP du signal émis
A digital TV transmission must be « Quasi Error Free » (QEF):

\[ \text{TEB} < 10^{-10} \]
Satellite Digital Video Broadcasting : DVB-S

Physical layer

Codage source et multiplexage

Mux Adaptation and energy dispersal

Outer code

Interleaver

Inner code

Mapper

Shaping filter

To RF Satellite Channel

AWGN channel with non linearities

Train transport

Program Information

Video Coder

Audio Coder

Data Coder

ES

PES

PCR

Program 1

Video Coder

Audio Coder

Data Coder

STC 1

Program N

Video Coder

Audio Coder

Data Coder

STC N

Reed Solomon
RS(204,188, t=8)

Forney Convolutional Interleaving

Convolutional code (7,1/2)

QPSK Modulation

SRRCF

\( \alpha = 0.35 \)

Physical Interface

\( f \)

\( H(f) = \begin{cases} 
1 & \text{pour } |f| < f_N(1-\alpha) \\
\frac{1}{2} + \frac{1}{2} \sin \left( \frac{\pi}{2f_N} \left[ \frac{f_N - |F|}{\alpha} \right] \right) & \text{pour } f_N(1-\alpha) \leq |f| \leq f_N(1+\alpha) \\
0 & \text{pour } |f| > f_N(1+\alpha)
\end{cases} \)
→ **Digital Communications**, J. G. Proakis, Mac Graw Hill Book Cie

→ **Telecommunications system engineering**, Lindsay and Simon, Prentice Hall

→ **Digital communication by satellite**, J.J. Spilker, Prentice Hall

→ **Digital Video Broadcasting (DVB)**: Framing structure, channel coding and modulation for 11/12 GHz satellite services, norme ETSI EN 300 421.

→ **Digital Video Broadcasting (DVB)**: User guidelines for the second generation system for broadcasting, interactive services, news gathering and other broadband satellite applications (DVB-S2), norme ETSI EN 102 376.